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Asset Liability Management for pension funds:
Assessment of common practices

by

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In loving memory of my father.
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The paper that lies before you is the result of a 6-month long internship at Synopsis Asset Management in the context of my master studies at the University of Lausanne. The topic of Asset Liability Management (ALM) was chosen because the ensuing strategic asset allocation provides the investment framework of institutional asset managers such as Synopsis. Pension funds, hit hard by the economic crisis, have been lately questioning the need for an ALM study, and Synopsis also believed that the strategic asset allocation with its tight leeways could hold an asset manager hostage at a time when fast investment decision making is needed the most. Hence Synopsis expressed the desire to investigate the mechanisms behind the determination of the strategic asset allocation as well as possible improvements or alternatives to current ALM studies.

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Aminata Diara Wane, Lausanne, August 29, 2011.
ABSTRACT

This research paper came to be due to the disillusionment of Swiss pension funds with their ALM after the economic crisis hit in 2008. We broke down the ALM process to bridge the clear gap between the mechanisms of ALM and the interpretation of its results by pension funds. In the process, we were able to point out certain inconsistencies with the way the process is carried out: overly optimistic assumptions and as a result the exclusion of shock scenarios in the simulation, as well as the difference between economic value and economic structure of liabilities, the latter conditioning the use of a surplus instead of an asset-only optimization model. We warned against rushing towards seemingly revolutionary methods such as LDI - an extension of ALM at best, although we gave credit to its interesting reinterpretation of the strategic asset allocation as a two-level investment framework with the LHP as the portfolio the pension fund has to hold and the PSP as the additional portfolio the pension fund should hold to generate performance. We suggested looking in this direction for a dynamic strategic asset allocation based on a building block investment model.

Cette recherche est née du désillusionnement des caisses de pensions par rapport à leurs études de Congruence Actifs-Passifs (CAP ou ALM) au lendemain de la crise financière de 2008. Nous avons analysé les étapes de la démarche ALM en espérant réduire le fossé entre l’interprétation des résultats de cette démarche et les mécanismes qui la constituent. Nous avons pu mettre le doigt sur quelques incohérences du procédé derrière cette étude, à savoir des hypothèses trop optimistes, des scénarios économiques qui donc n’incluent pas de grands chocs et la différence entre la valeur économique et la structure économique des engagements, cette dernière conditionnant l’utilisation d’un modèle d’optimisation du surplus ou des actifs. Nous nous prononçons aussi sur le LDI, qui n’est certes pas la nouvelle trouvaille pour laquelle certains essaieraient de la faire passer. Par contre, nous trouvons intéressant son approche d’une allocation stratégique à deux niveaux, avec le LHP comme portefeuille que la caisse de pension devrait tenir, et le PSP comme portefeuille additionnel pour générer de la performance. Nous suggérons de creuser dans cette direction pour une allocation stratégique dynamique basée sur un modèle d’investissement à plusieurs niveaux.

**Keywords:** Asset liability management, ALM, liability driven investment, LDI, strategic asset allocation, Swiss pension funds, LPP, investisseurs institutionnels, prévoyance professionnelle, 2ᵉ pilier.

**JEL classification:** C00, G00
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INTRODUCTION

Asset Liability Management (ALM) is a complex term, which has evolved tremendously in the past few decades and which, to this date, can mean different things to a lot of people. Its interpretation can be as broad as the definition provided by the Society of Actuaries in their most recent ALM Specialty guide: "ALM is the practice of managing a business so that decisions and actions taken with respect to assets and liabilities are coordinated." They continue on by describing ALM as "the ongoing process of formulating, implementing, monitoring and revising strategies related to assets and liabilities to achieve an organization's financial objectives, given the organization's risk tolerances and other constraints". In that sense ALM would be to some pension funds an internal risk management process, providing a unique global view of the risks the pension fund is exposed to, recommending ways to handle those risks, should they materialize, and even giving the ability to test the long-term incidence of desired policy changes. In contrast, the Swiss Federal Insurance Office describes ALM as a method of management which guarantees that the strategic asset allocation of the pension fund matches the structure of its liabilities. Bringing the asset allocation aspect to the forefront of ALM has led – perhaps wrongly so – many Swiss pension funds to reducing the whole process to an outsourced study, aimed at producing a strategic asset allocation that theoretically guarantees the solvability of the pension funds with some high level of confidence. With the latter approach in mind, it is only natural that fingers pointed at ALMs, when 59.3% of pension funds reported being underfunded as of 31.12.2008, i.e. in the midst of the financial crisis. In fact, 15.2% of registered private Swiss pension funds reported a funding ratio below 90%, while 44.1% reported a funding ratio in the 90-99% range1(see [24]). In comparison, only a total of 3.9% had reported being underfunded at the end of 2007. However, before disqualifying ALMs as completely useless, Swiss pension funds need to have a clear understanding of the goals they set out for their ALMs, how those goals translated into the ALM assignment and whether the mechanisms of the ALM can realistically lead to the achievement of those goals. There has been a lot of literature on the advantages and disadvantages of various economic or mathematical models used in ALM, but very little step by step breakdown approach to an ALM study. This is the approach we want to take in this paper, as it facilitates the transition from theory to practice, in that it puts the

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1 These statistics were provided by the Swiss Federal Office of Statistics (OFS). Were taken under consideration only private, registered autonomous and semi-autonomous pension funds: 1407 in 2008 vs. 1435 in 2007. Since regulation is less stringent with public pension funds when it comes to underfunding, we did not include them in the statistics.
models studied in the academic literature in context. Throughout the paper we will look at common practices in Switzerland, analyze their implications in the ALM study and introduce, when relevant, other models found in academic literature or in the practice in other countries.

We start out by putting ourselves in the shoes of an ALM practitioner who needs to explain the intricacies of an ALM study to the board of trustees of a Swiss pension fund. The following figure (FIG. 1) illustrates the axes around which our analysis will revolve.

![Diagram](image)

**FIG.1:** Step by step breakdown of the ALM process

At each step, we present real situation examples, which are intended to enhance the comprehension of the discussed models.

We follow this overview of ALM practices with a comprehensive look at the challenges pension funds face nowadays, and which of those challenges are realistically addressed through ALM studies.
PART I: Breakdown / analysis of the stages of an ALM study

First and foremost, Asset Liability Management always involves some sort of modelling of the assets and liabilities of the pension fund for a given time horizon (usually of at least 10 years). In this paper, we will assume a time horizon of 10 years. Given the long-term nature of this projection, there are many elements of risk, whose variability over the chosen time horizon can jeopardize the health of the pension fund. Too often, when we talk about risks in the context of ALM, the focus is put on general risks such as credit risk or interest rate risk. To determine to what extent those risks have been analyzed, the mathematical and economic models are the ones being scrutinized. Seldom do we offer a critical look at the assumptions in their format and in their content. In this section, we will reflect on the reasoning that should accompany the decision making process when it comes to choosing assumptions and on the implications of common practices in Switzerland.

1 Determination of actuarial and financial assumptions

1.1 Actuarial Assumptions

In modelling the liabilities of a pension fund, the actuary makes a lot of assumptions about the events that can happen in the insured's lives over a period of 10 years. In Switzerland, pension funds provide disability, survivorship and old age benefits:

- In case of disability before the retirement age, the insured as well as any of their children under 18 years old (25 years if still at school) receive benefits.
- In case of death before the retirement age, the spouse of the insured and any of their children under 18 years old (25 years old if still at school) receive benefits.
- In case of retirement, the insured as well as any of their children under 18 years old (25 years if still at school) receive benefits.
- In case of death after retirement, the spouse of the insured and any of their children under 18 years old (25 years old if still at school) receive benefits.

In addition, the insured are given the option of receiving a lump sum instead of an annuity at the time of retirement as well as withdrawing a part of their pension to invest in the purchase or renovation of their primary housing. In case of withdrawal, the insured is entitled at the very least to the funds they have contributed to their retirement so far. In case of divorce, the pension fund may have to transfer some of the insured's accrued benefits to the pension fund.
account of the former spouse, since by law they have to share the benefits accrued during their marriage equally.

From the list above, we can see that we have to consider a lot of factors:

1. Death or disability probabilities of the insured,
2. early retirement of the insured,
3. probability that the insured is married/will marry in the period of time considered,
4. age of the spouse,
5. life expectancy of the spouse (to estimate how long the survivor pension would have to be paid for),
6. probability of having children at the time of invalidity, death or retirement (how many on average and what would be their age?),
7. probability that the children, if between 18 years old and 25 years old, are still at school,
8. the turnover rate at the company.

Most Swiss pension funds base their calculations on more or less modified versions of common actuarial tables:

- EVK 2000, recently updated to EVK 2010, which is compiled by the Federal Insurance Fund,
- VZ 2005 compiled by the Canton of Zurich, or
- LPP 2005, updated to LPP 2010 compiled by two consulting firms, Aon Hewitt and LCP Libera.

The choice of the most suitable table for a given pension fund is left to its board of trustees, keeping in mind that differences between the tables can be substantial depending on the situation of a pension fund. We will note for example that the EVK 2000 table is a model without reactivity – once a person is disabled, he cannot become active again. In addition, life expectancies have been found to be severely underestimated. There are also generalizations inherent to the construction of life tables: for every insured at a given age, only the age that the spouse would have on average would be considered, and similarly, only the age of the child on average would be considered - with the assumption that the insured only has one child who would be entitled to benefits. Hence the tables can only more or less describe the reality of the pension fund; either modifications are brought directly to the table (numerical changes in the probabilities) or provisions are set aside for risks that the board of trustees

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2 A role to be reinforced by the Swiss structural reform, scheduled to come into effect as of July 1st, 2011.
anticipates were underestimated by the chosen table. We often find provisions for longevity risk in the balance sheet of Swiss pension funds, but other provisions may also be set up, for example provisions for disability risk for pension funds of employees in the skilled craftsmanship line of work. Parameters such as the withdrawal rate and the entrance rate are often considered to be more company-specific and hence are left to the board of trustees to determine. Some companies may find them to be correlated with economic parameters (described in the following section) and should make sure to take that fact into account when coming up with their assumptions. We need to acknowledge here the static nature of most actuarial tables used in Switzerland. The use of provisions helps to amortize the effect of having underestimated the actuarial parameters. Making those values fluctuate randomly, but within a reasonable range, through the use of generational tables for example, would provide more realistic results; however the gain in accuracy may not justify the cost of such added complexity into the model. The bottom line is that the Board of Trustees needs to make a decision on their tolerance level to actuarial risk and ensure that the modelling of those actuarial parameters matches the specifics of their pension fund and the risk tolerance level agreed upon.

1.2 Financial Assumptions

In the use of actuarial tables, we also have to realize that beyond the calculation of probabilities, there are some financial considerations taken in order to compute the present value of the annuities and lump sums. Using these results, we can compute the provisions for the active insured and for the pensioners, as they appear under the section of liabilities on the pension fund's balance sheet. Those financial considerations are:

- The technical interest rate

  The technical interest rate is basically the rate at which the liabilities of current and future pensioners are discounted. Until now, the board of trustees could determine the technical interest rate for their pension fund. Per the DTA 4 directive, which should come into effect on January 1st 2012, the chamber of Swiss actuaries will provide a yearly reference technical interest rate, based on the LPP 2005 index by Pictet LPP-25 plus as well as the 10-year yield of federal bonds (Chamber of Swiss Actuaries [10]). This reference rate is always to be higher than the 10-year yield rate of federal bonds.
obligations but below 4.5%. Swiss pension funds can go over the reference technical interest rate by no more than 0.25%. With this in mind, we can see that assumptions about technical rates have to go hand in hand with the assumptions about inflation as well as the evolution in the market – an aspect that should be addressed for each economic scenario.

- **The salary growth rate**
  Future pension liabilities are often computed on the basis of the salaries at the end of the career. While the company may have its own scale based on years of service, experience and educational background, we must not forget to address inflation-related salary raises.

So far, we have only mentioned financial parameters that have an impact on the evaluation of liabilities. Let us have a look at the parameters that have more of an impact on the asset side:

- **The contribution rate**
  According to a recent study published by Credit Suisse (2009), 9/10 of Swiss pension funds have defined-contribution plans, making the contribution one of the more stable financial parameters due to its predictability. However, depending on the pension fund's risk-sharing profile, its projection can become quite tedious: some pension funds have policies such that the contribution rates of the company and employees vary based on the funding ratio of the pension fund. Hence an accurate ALM study should reflect this fact in the modelling of the pension fund from one year to the next.

- **The return rates of the asset classes**
  A first step consists of deciding on the asset classes that we are allowed to have in the institutional portfolio. The articles 53, 54 and 55 of the Swiss law LPP OPP2 provide general guidelines; let us note in particular the boundaries on the allowed asset classes: at most 50% on mortgage securities, 50% on actions, 30% on real estate (of which, at most 1/3 is on foreign real estate), 15% on alternative investments, 30% in foreign currency without hedging of the exchange rate risk. In practice, these regulations are seldom followed to the "T", since the second paragraph of the article 59 LPP OPP2 allows for "violations" of the above limits under certain circumstances left to the discretion of the surveillance authority. We will note, for example, that commonly used reference indices Pictet LPP Plus have at their neutral point at least 40% invested in foreign currency. Still, even when board of trustees take into account the limitations
set by these regulations, they are left with quite a lot of freedom, when it comes to the specifics of the asset classes in the portfolio. The choice of the types of assets used must be done with the following criteria in mind:

1. low correlation with other asset classes,
2. high correlation with liabilities, and
3. high Sharpe ratios (reward to variability ratios).

It is also misleading to think that including as many asset types as possible will lead to better portfolio optimization results, as in fact too much diversification will dilute the performance.

A second step consists of gathering the historical return rates and covariances of the specific asset classes in the mix decided on by the board of trustees. Here also, the board of trustees must familiarize themselves with the market indices used to represent the types of asset classes selected, and agree on their calculation methods and on the fact that their sub-indices are varied enough to represent the whole market for that asset type. Of course, we make the implicit assumption that historical observations are more or less predictive of the future, but we have to be careful here. It is common practice to exclude shock data, which means that crises are not taken into account, at least not from the hypotheses point of view.

A third step consists of making a projection of the average return rates and covariances for the asset classes over the next 10 years. This projection is primordial and should be done carefully, as it alone serves as the financial framework for the portfolio optimization. Particularly, it should reflect the convictions regarding the long term evolution of the market of the board of trustees, the asset managers and other knowledgeable financial advisors.

- Inflation

At this stage, we can also note the fact that Swiss pension funds are not required to index pensions due to inflation - only disability and survivorship benefits are indexed. Hence, for most pension funds\(^4\) the consequences of inflation are rather indirect, in that they would ensue from the correlation between inflation, the evolution of the market and other financial parameters such as the technical interest rate and the salary growth rate.

\(^4\) Pension funds having insureds with higher probabilities of disability would have to take this into account.
In this section, we have identified the inputs in the development procedure of the assets and liabilities of the pension fund. Assuming that we have collected all those inputs, how do we go from there to producing an economic value for the assets and the liabilities? The next two sections will provide answers to this question.

2 Developing the liabilities

We start out with figuring out the demographic structure of the pension fund by using the push-pull Markov model (see [7]).

2.1 The Push – Pull Markov Model

The push aspect of the model consists of classifying the members by age, gender, salary, years of service as well as any other characteristic of the members coming into account when thinking of the salary scale in the company, and by applying a matrix of transitional probabilities with respect to survival, disability and retirement, we "push" the members from their "old" state to their "new" state.

![Diagram](image)

**FIG. 2: Push Markov model for a pension fund**

The push Markov model hence gives us a precise idea of the status of each current member of the pension fund at time $t$ based on the actuarial tables and on the occupation of the members.
The pull Markov model, on the other hand, allows for the incorporation of the expected development in the size and structure of the work force in the pension fund's underlying company (-ies). For example, if the underlying company wishes to keep the same size but aims for a younger structure, then we have to make sure that each voluntary withdrawal or death will be compensated with a new entrant, whose age, on average, will be lower than that of those who are no longer members of the pension funds. Characteristics of the new entrants such as their salaries will be in line with the current members in the same gender, age and status class.

Now that we know our liabilities in demographic terms, we are ready to talk about the liabilities in economic terms.

2.2 Cash value of the liabilities

In the literature, we find two acceptable ways of computing the cash value of the liabilities: they are found under the terms of Accumulated Benefit Obligation (ABO) and Projected Benefit Obligation (PBO). The ABO answers the question of what the liabilities would be at time \( t \) if the pension fund were to close exactly at that time. The computation is simplified in the sense that all benefits are computed at the level of the salaries at time \( t \). In contrast, the PBO takes into account the salary growth, in the sense that benefit calculations are based on what we believe the end-of-career salaries will be. In that regard, the PBO is much more realistic than the ABO. Intuitively, the PBO is often higher than the ABO - at the very least, they are equal. In Switzerland, the PBO seems to be the norm as required under Swiss GAAP RPC 26. We find it important to underline this fact, as those same accounting rules should apply for the computation of past and future liabilities. As mentioned in section 1.2, the salary growth rate formula should both reflect the salary scale of the underlying company as well as the possible impact of inflation.

Now that we have end-of-career salaries figured out, we can extract the technical interest rate from the term structure of interest rates. Quick recall: We will use the technical interest rate to compute the present value of the accrued benefits. Therefore, based on the pension deal, we are ready to compute:

1. An approximation for the benefits to be paid at time \( t \).
2. The mathematical reserve for the actives⁵.
3. The mathematical reserve for the pensioners⁶.
4. The technical provisions for longevity risk and for any other actuarial fluctuation risks judged necessary by the pension fund such as disability and survival risks (see actuarial assumptions).
5. Any foreseeable fees such as liabilities resulting from reinsurance contracts (often for disability and survival).

The benefits and the foreseeable fees are actual negative cash flows: they are money spent at time \( t \). Mathematical reserves and technical provisions are money that we need to keep in mind does not belong to us. They guarantee that we will be able to make those cash payments, when their time to be paid comes.

To accompany our reflection, let us have a look at the following two figures typically provided in ALM reports (in fact, they were provided to us by an anonymous Swiss pension fund). This pension fund has a defined–contribution pension plan.

![Graph showing predicted annual cash flows](image)

Source: ALM report of anonymous pension fund

Years

**FIG. 3:** Predicted annual cash flows not taking into account the asset return

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⁵ The active insured's accrued benefits, known as "libre passage" in Switzerland
⁶ The retired insured's accrued benefits minus pensions already paid, known as "Avoir de vieillesse" in Switzerland.
From FIG. 3, we can see that this pension fund is doing well with respect to cash being readily available to pay off expenses, since the contributions alone cover the cash outflows. However the surplus is decreasing significantly over the 10 years. This means that, soon after those ten years have passed, contributions may not be enough to foot all the expenses, and the pension fund would have to draw from its actual assets. This future need for liquidity may have to be taken into account - if not now, in a few years – in the composition of the portfolio: for example, if the pension fund is reticent to hold too much liquidity in their portfolio, they could plan ahead and use investment instruments, whose maturity date [or coupon payment date] will coincide with the time those expenses are due.

The second figure (FIG. 4) gives some explanation of the cash flows observed in FIG. 3. The mathematical reserves for the pensioners almost triple over the 10 years: the pension fund has quite a few members retiring soon, hence the increase in the expenses observed in FIG. 3. Those soon-to-retire members also participate in the increase of the contributions, because contributions are higher towards the end of the members’ career. We also notice the growth of the mathematical reserves for the actives, which doubled over the 10 years, also explained by those same members: as they get closer and closer to retirement, their accumulated benefit credit is much higher if they were to walk out then than if they had walked out earlier. The
increase in provisions for the longevity risk is also comprehensible: having more and more newly retired members, the pension fund needs a little more cushion in case those turn out to live for a long time.

Not surprisingly, we have only talked about retired and soon-to-retire members. Very young plan members have lesser influence on the two figures presented above: their contributions and mathematical reserves are much smaller on a one–to–one comparison with older plan members. However those young members will be the mid-career members of ten years from now, and if the company turnover at that level is low, they can significantly alter the risk profile of the pension fund: providing enough liquidity to pay off expenses without going into the portfolio, and allowing the pension fund to be more risk-friendly because of the longer investment horizon. While knowing how much you are going to owe should naturally give you some insights on how you are going to manage your wealth, keeping the age structure in mind always keeps you a step ahead in terms of investment risk management.

In this section, we discussed how we go from actuarial and financial assumptions to the liabilities as they would appear on the pension fund's balance sheet. The bottom line is that we should:

1. Ensure that there is enough liquidity to pay off those cash payments at the moment they are due; if this fact cannot be guaranteed by the contributions, it can have an impact on the investment strategy.
2. Ensure that the total value of our assets, once we pay off everything that we owe for that year in cash, at the very least covers our mathematical reserves and technical provisions,
3. Not limit ourselves to just economic values. The age structure is a valuable gauge of the future liabilities and as such it keeps us a step ahead in our quest of being ready to meet those liabilities.

3 Modelling the assets

The wealth of the pension fund is mainly determined by the present value of its portfolio of assets, the choice of which can be based on different criteria. We will clearly distinguish four methods here:
3.1 The duration matching method

This was the method of choice in the early days of ALM. Asset Liability Management is born out of a desire to minimize the mismatch between assets and liabilities in the event of unfavourable interest rate changes. This particular method consists of identifying bonds that best match the characteristics of our liabilities over the period under consideration, and make sure that we select assets such that their duration matches that of the bonds chosen above. This clearly works best for short term management, as liabilities would have a shorter duration, and when bonds are essentially the vehicles of investment. Besides this method can only be used with a starting funding ratio of at least 100%. A method that is mostly used by banks and insurance companies, it is applicable to few pension funds such as those that only have pensioners (see [3]).

3.2 The asset-only framework

Theoretically speaking, this method does not fall under asset liability management, but practically speaking, it is arguably the most widely used method. Why? Because of its simplicity. Based on Markowitz portfolio theory, it is relatively easy to come up with an efficient portfolio that minimizes the risk level for a preferred rate of return or that maximizes the rate of return for a preferred risk level.

We start out with our projection of the 10-year return rates and covariances for the available asset classes. We insist on the long term and static nature of this projection, which should answer the question of the expected average yield rate and risk level for each asset class, if we were to average over 10 years. Thus, we only need the historical expected values and covariances, unless we believe that those are only partially representative of the future, in which case they can be adjusted to meet the views of the board of trustees of the pension fund. What follows is a simple mean-variance optimization problem.

We will again use the data provided by an anonymous pension fund to illustrate the steps behind a simple mean-variance optimization problem. The indices and risk and return statistics are available in Appendix A.

Finding efficient portfolios using Markowitz modern portfolio theory consists in solving the following equation:
\[
\min_{x_i} \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij},
\]

with the following constraints:

(A) \[\sum_{i \in n} x_i = 1,\]

(B) \[\sum_{i \in n} \mu_i x_i = \text{some desired portfolio mean return } \mu_p,\]

where \(n\) is the number of assets, \(x_i\) is the weight of asset \(i\) in the portfolio and \(\mu_i\) is the mean return of asset \(i\).

Other constraints on the asset weights can be added, but let us keep in mind that constraints can significantly alter the efficient frontier. Therefore those restrictive decisions should not be taken lightly. Using the assumptions presented in Appendix A, we constructed three efficient frontiers, each corresponding to a different set of constraints: a no short-sale constraint (i.e. \(x_i \geq 0\) for all \(i\)’s) since shortselling is generally not possible for pension funds, the OPP2 limits constraint and arbitrary constraints based on what we assume are convictions issued by the Board of trustees. The results are presented on the figure below (FIG. 5).

![Efficient frontiers obtained with different sets of constraints](image)

**FIG. 5: **Efficient frontiers obtained with different sets of constraints

We found also interesting to look at the impact of the constraints on the weights of the portfolio. The table below (TAB. 1) offers a comparison of the asset weights in the global minimum variance portfolios and for the portfolio with return \(\mu_p = 5\%\).
In terms of optimality, any of the portfolios on the efficient frontier is eligible. So how would the pension fund go about choosing the one that will be used as the strategic asset allocation? The preliminary decision phase consists of identifying a few portfolios of interest (usually one to five portfolios). A portfolio can be of interest to a pension fund because it satisfies either the risk tolerance or the desired return level of the pension fund, or because it has a high Sharpe ratio. For example, if the portfolio of assets the pension fund currently holds is not on the efficient frontier, the corresponding optimal portfolio would be of interest, assuming the risk profile of the pension fund has not drastically changed.

However, one could also go about making this choice in a different way, using a method that has the merit of somewhat including the liabilities into the decision making process.

Assuming the following future surplus or shortfall pattern for a pension fund (TAB. 2), we want to know what return will ensure that we will have the necessary wealth to cover our liabilities at the end of the 10 years. The answer for this example is 4.58%.

<table>
<thead>
<tr>
<th></th>
<th>No short sale</th>
<th>OPP2</th>
<th>Arbitrary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>Minimum</td>
<td>Minimum</td>
<td>Minimum</td>
</tr>
<tr>
<td>Variance</td>
<td>Variance</td>
<td>Variance</td>
<td>Variance</td>
</tr>
<tr>
<td>Portfolio</td>
<td>Portfolio</td>
<td>Portfolio</td>
<td>Portfolio</td>
</tr>
<tr>
<td>with return</td>
<td>with return</td>
<td>with return</td>
<td>with return</td>
</tr>
<tr>
<td>of 5%</td>
<td>of 5%</td>
<td>of 5%</td>
<td>of 5%</td>
</tr>
<tr>
<td>Return</td>
<td>1.520%</td>
<td>5%</td>
<td>1.525%</td>
</tr>
<tr>
<td>Risk</td>
<td>0.199%</td>
<td>3.69%</td>
<td>0.198%</td>
</tr>
<tr>
<td>Weights</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>99.08%</td>
<td>0</td>
<td>99.08%</td>
</tr>
<tr>
<td>Bonds CHF</td>
<td>0.44%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Foreign Bonds</td>
<td>0.08%</td>
<td>0</td>
<td>0.08%</td>
</tr>
<tr>
<td>Mortgages CHF</td>
<td>0</td>
<td>18.72%</td>
<td>0.44%</td>
</tr>
<tr>
<td>Bonds CHF</td>
<td>0</td>
<td>10.14%</td>
<td>0</td>
</tr>
<tr>
<td>Equity CHF</td>
<td>0</td>
<td>12.52%</td>
<td>0</td>
</tr>
<tr>
<td>Equity World</td>
<td>0.04%</td>
<td>0.75%</td>
<td>0.04%</td>
</tr>
<tr>
<td>Real estate CHF</td>
<td>0.33%</td>
<td>21.45%</td>
<td>0.33%</td>
</tr>
<tr>
<td>Swiss real estate fdp</td>
<td>0</td>
<td>28.07%</td>
<td>0</td>
</tr>
<tr>
<td>Foreign real estate fdp</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hedge funds</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Commodities</td>
<td>0</td>
<td>1.67%</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0.02%</td>
<td>6.68%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

TAB. 1: Comparison of asset weights in two optimal portfolios depending on the type of constraints
Let us note that this critical return only ensures that the pension fund will be fully covered in 2018. The in-between funding ratios are not at all taken into account. In fact, if our portfolio were to have a steady return of 4.58% for the 10 years, the pension fund will remain underfunded for the next nine years. If the pension fund wanted to set its critical return at a rate that will ensure the pension fund is fully funded for each of the following year for the next 10 years the rate would have to be of 8.875%, given that the pension fund starts out with a funding ratio of 94%. A return of 4.9% would ensure that the pension fund is fully funded as of year 5.

<table>
<thead>
<tr>
<th>Year</th>
<th>Assets</th>
<th>Surplus (Shortfall if negative)</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>386.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>29.8</td>
<td>452.5</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>28.9</td>
<td>499.5</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>27.8</td>
<td>547.8</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>25.4</td>
<td>596.1</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>25.4</td>
<td>646.9</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>22.5</td>
<td>697.3</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>22.0</td>
<td>749.6</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>22.5</td>
<td>805.2</td>
<td></td>
</tr>
<tr>
<td>2017</td>
<td>20.8</td>
<td>861.4</td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>18.2</td>
<td>917.8</td>
<td></td>
</tr>
</tbody>
</table>

TAB.2: Assets, Liabilities and Cash flow of a pension fund

The bottom line is that deciding on a critical return narrows down the choice of eligible portfolios, as all the portfolios with lower return would no longer be considered - and reflects in concrete terms the direction in which the pension wishes to be heading. Now that we have a set of chosen portfolios, we will later test the magnitude of the impact favourable and unfavourable, yet realistic economic scenarios, will have on our pension fund when holding each of those portfolios. We will develop this further in section 4.

For now, a question lingers: Aren't liabilities also part of the optimization problem? So far, we have only considered them as the target wealth for the pension fund. By doing so, we assume that assets and liabilities are not affected by the same market-related factors. This is an erroneous assumption on many levels. For example, if the interest rate of 10-year bonds were to decrease today, this would be good news for assets, as they would appreciate, but bad news
for the liabilities since the discount rate is lower, leading to higher present values. If on the other hand the interest rate of 10-year bonds were to increase, this would be bad news for the assets, as they would depreciate, but good news for the liabilities since the discount rate is higher, leading to lower present values. A pension fund with just the right amount of long-term bonds may not feel the impact of this change in interest rate much – in particular the appreciation of the bonds will make up for the increase of the present value of liabilities, while ensuring that in the case of the depreciation of both assets and liabilities, the congruence between them is still maintained. The relationship between liabilities and the market are much more complicated than the simplified example given above, but we do realize the necessity of including the economic dimension of liabilities in the choice of a strategic asset allocation. In the following section, we go over a methodology that, in theory, allows us to achieve this goal.

3.3 The surplus optimization method

We will present the surplus optimization method as introduced by Sharpe and Tint in their 1990 paper: "Liabilities: A new approach" (see [21]). This method is interesting in the sense that it is an extension of the asset-only model, with the slight modification that we are not doing a mean–variance optimization on the return of the portfolio, but rather on the return of the pension fund’s surplus. We define the surplus as:

\[ S_0 = A_0 - L_0, \]

where \( S_0, A_0 \) and \( L_0 \) represent respectively the economic value of the surplus, assets and liabilities today. As such, these values are known. Let \( \bar{S}_1, \bar{A}_1 \) and \( \bar{L}_1 \) be the respective economic values next year, with the tildes representing the fact that those values are unknown. Analogously, we can state that:

\[ \bar{S}_1 = \bar{A}_1 - \bar{L}_1. \]

Pension funds are concerned with maximizing next year’s surplus with respect to their fortune today, which translates to the following equation:

\[
\max \frac{\bar{S}_1}{A_0} \Rightarrow \max \left( \frac{\bar{A}_1}{A_0} - \frac{\bar{L}_1}{A_0} \right).
\]

We can introduce the more familiar notion of the funding ratio by multiplying the last term by \( \frac{L_0}{L_0} \). We then get:
Using return notation, this means that:

\[
(1 + \tilde{R}_A) - \frac{L_0}{A_0} (1 + \tilde{R}_L).
\]

You would notice here that \(\frac{L_0}{A_0}\) is the reciprocal of the pension fund's funding ratio. This means that the weight we want to give to the liabilities with respect to the assets in our surplus optimization depends on the pension plan's current situation. This makes a lot of sense!

Rewriting the above formula, we get:

\[
\max \left( \left[ -\frac{L_0}{A_0} + \tilde{R}_A - \frac{L_0}{A_0} \tilde{R}_L \right] \right).
\]

The first bracketed expression does not involve any uncertain terms; hence it is of little interest to us for the purpose of decision making. We can thus concentrate on the second part, which, for convenience reasons, we shall name \(\tilde{Z}\), such that:

\[
\tilde{Z} = \tilde{R}_A - \frac{L_0}{A_0} \tilde{R}_L.
\]

So far this reasoning is very intuitive.

At this point, we return back to Markowitz's mean-variance optimization model, in that we want to minimize \(\text{Var}(\tilde{Z})\) (or maximize \(E(\tilde{Z})\)). Using properties of expectation and variance, we obtain:

\[
E(\tilde{Z}) = E\left( \tilde{R}_A - \frac{L_0}{A_0} \tilde{R}_L \right) = E\left( \tilde{R}_A \right) - \frac{L_0}{A_0} E\left( \tilde{R}_L \right),
\]

and

\[
\text{Var}(\tilde{Z}) = \text{Var}\left( \tilde{R}_A - \frac{L_0}{A_0} \tilde{R}_L \right) = \text{Var}\left( \tilde{R}_A \right) - 2 \frac{L_0}{A_0} \text{Cov}\left( \tilde{R}_A, \tilde{R}_L \right) + \frac{L_0^2}{A_0^2} \text{Var}\left( \tilde{R}_L \right).
\]

Again, we can simplify this problem, since the decision we make on the asset allocation does not affect the return of the liabilities. So in terms of the portfolio choice decision, we can conclude that:

- maximizing \(E(\tilde{Z})\) is equivalent to maximizing \(E(\tilde{R}_A)\), and
minimizing $\text{Var}(\tilde{Z})$ is equivalent to minimizing $\text{Var}(\tilde{R}_A) - 2\frac{L}{A_0}\text{Cov}(\tilde{R}_A, \tilde{R}_L)$.

Hence, this method differs from the asset–only framework by the term $\text{Cov}(\tilde{R}_A, \tilde{R}_L)$, whose weight into the optimization is defined by the current funding ratio of the pension fund. The difficulty we face in practice arises from the determination of the liability return and of its covariance with the asset classes. We could distinguish two main methods in the literature, although variations of those methods abound.

The first method, proposed by Mark Kritzman (1990), consists of:

- recalculating the historical liabilities of the pension fund under the present actuarial valuation methods, but with the use of past interest rates: this will produce a time series of the liabilities,
- computing the return and variance of the liabilities,
- regressing the liability returns against those of bonds, stocks and short-term securities to compute the correlation between the assets and liabilities.

This method is rather tedious and requires access to extensive historical data of the pension fund. And could we assert that past liabilities are representative of the future liabilities? Probably not. However, we are looking for the correlation between the assets and liabilities, and the past correlation can well be representative of the future correlation, provided that liabilities are computed on the very same basis as they would be in the future. Then again, such detailed, historical data may be hard to come by.

The second method found in the literature follows a more straightforward approach to incorporating the economic dimension of the liabilities. Aaron Meder and Renato Staub (2007) suggest a decomposition of the pension liabilities such that the market related exposures can be mimicked with certain asset classes. The following table (TAB. 3) summarizes their result:

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Market-related exposures</th>
<th>Liability Mimicking Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pensions</td>
<td>Term Structure</td>
<td>Nominal bonds</td>
</tr>
<tr>
<td>Accrued benefits for the active</td>
<td>Term Structure</td>
<td>Nominal Bonds</td>
</tr>
<tr>
<td>Future pensions for the active driven by the future wage growth</td>
<td>Inflation, Growth, Term structure</td>
<td>Real Rate Bonds, Equities, Nominal bonds</td>
</tr>
</tbody>
</table>

TAB.3: Mimicking the liabilities with certain asset categories
Meder and Staub first identified the fundamental factors of those asset categories: real risk-free rate, inflation proxied by the consumer price index (CPI), economic growth, equity premium, nominal bonds premium and real bonds premium. With the corresponding historical data, they determined the risk and correlation of those factors. Their next step was to compute the sensitivities of assets and liabilities with respect to changes in the underlying factors. The elements that were highly sensitive to changes in the same underlying factors were highly correlated, and similarly for the reverse relationship. The resulting correlations are summarized in TAB. B.1 (Appendix B). Meder and Staub did not present this work with a mean-variance optimization in mind. However, we could incorporate their findings in this framework by going from their liability decomposition to the notion of liability return $\bar{R}_L$. We will use the same correlation and risk measures found by Meder and Staub (TAB. B1). For simplification purposes, foreign Bonds CHF will be arbitrarily used as a proxy for real rates bonds, noting that in practice we will clearly need a more thorough analysis of the asset categories available to Swiss institutional investors, of the decomposition of the liabilities, which is specific to the pension fund, and of the correlation between those assets and liabilities. For assumed liability decomposition weights of respectively 0.8, 0.1 and 0.1 and current funding ratio of 0.94, we find the risk of $\bar{R}_L$ and the covariances $\text{Cov}(\bar{R}_j, \bar{R}_L)$ for each asset class as presented on TAB. B2 and TAB. B3 in Appendix B.

The two methods described above demonstrate the kind of reasoning and the tedious work that accompany the determination of $\bar{R}_L$ and $\text{Cov}(\bar{R}_j, \bar{R}_L)$. Once we have these values, we are again only left with a mean-variance optimization problem. More specifically, we are going to minimize

$$\min_{x \in \mathbb{R}^n} \left\{ \sum_{i,j} x_i \sigma_{ij} x_j - \frac{1}{f_0} \sum_i \sigma_{ii} x_i \right\}$$

where $\sigma_{ij} = \text{cov}(\bar{R}_i, \bar{R}_j)$, $\sigma_{ii} = \text{cov}(\bar{R}_i, \bar{R}_L)$ and $f_0$ is the current funding ratio of the pension fund, with the following constraints:

(A) $\sum_i x_i = 1$, 

(B) $\sum_i \mu_i x_i \geq$ some desired portfolio mean return $\mu_p$.

---

7 To learn more about factor-based covariance matrix modelling: see [22].
Looking at the efficient frontiers (FIG. 6), it does not seem there is much change from the asset-only framework. If we look at the numbers however, we can see that for a return of 5%, the risk is higher for the surplus optimization results. Understandably, this increase is accounted for by the liability risk, which we count as zero in the asset-only model. This actually results in a slight horizontal shift of the efficient frontier to the right, but the biggest impact is actually on the composition of the optimal portfolios. In the results summarized in the table below (TAB. 4), we see that, in comparison to the asset-only model, the accent is shifted to the asset categories, which we decided earlier best mimicked the liabilities. We also calculated the surplus risk for each portfolio choice with a return of 5% (TAB. 4). We also notice that constraints have quite an impact on the surplus risk, especially for the arbitrary constraints, which imposed minimum values as well as maximum values on asset categories. The global minimum surplus variance portfolio, defined as the portfolio with the highest return and zero surplus return variance, hedges the market risks of the liabilities as much as is possible with the available asset categories. The portfolios of interest are again those that satisfy a certain level of risk or a certain level of return and have high Sharpe ratios and now we also have an added factor of decision in the form of the surplus risk of those portfolios. Albeit its intuitiveness and the fact that it has been researched and discussed as an extension of the Markowitz model for pension funds for quite some time now, there has not seemed to be a strong move towards the use of the surplus optimization model in recent years. Some of the reasons have been the fact that pension funds starting out underfunded are reticent to have such large amounts of money "sitting" in long bonds, or that nominal and real bonds with the desired duration can be hard to come by. Rolling short-rate investments, swaps and inflation swaps have been proposed as solutions, but the terms of these synthetic instruments are not always very clear, regulators are wary of derivatives – OPP2 laws allow a maximum of 15% on alternative investments, and of course we have to mention the need collaterals and the additional risk being incurred, notably the counterparty risk (see [9]). Nonetheless we found the surplus optimization model to be more suitable in terms of truly minimizing the probability of a mismatch between the assets and the liabilities, and as such, any efforts towards finding ways to improve its practicality would be most beneficial to pension funds.
FIG. 6: Comparison of efficient frontiers obtained with surplus and asset-only optimization
### 3.4 The LHP/PSP method (also known as modern/dynamic ALM, also known as LDI)

The two optimization methods described above both use the cornerstones of modern portfolio theory to identify the strategic asset allocation of a pension fund in the form of a single portfolio. Some practitioners in the field however believe that modern portfolio theory could provide further insight to ALM portfolio strategists. In that regard, Tobin's Two-Fund theorem (1958), Sharpe's ensuing Capital Asset Pricing Model – CAPM (1964) and Black's mutual fund separation theorem (1972) come to mind.
Let us first recall Tobin's Two-Fund theorem, which states that any mean-variance optimal portfolio can be rewritten as a combination of two portfolios: one delivering the risk free return and the other one, known as the market portfolio, delivering the risky return.

Black revisited this theorem with the assumption that there is no riskless asset. His conclusion is better known as the mutual fund separation theorem, which states that any mean-variance optimal portfolio can be rewritten as a combination of the market portfolio and a "zero-beta" portfolio, i.e. the minimum variance portfolio with zero covariance with the market portfolio. It is important to note here that the zero-beta portfolio is not on the efficient frontier.

Why are these separation theorems of importance to an ALM portfolio strategist? Well, think about it! In essence, these theorems allow us to rewrite an efficient portfolio as a combination of two other funds. What if we took one of those funds to be the Liability Hedging Portfolio (LHP) - hedging the liabilities, while the other one naturally becomes the Performance Seeking Portfolio (PSP) - seeking additional return? We would then go from looking at a pension fund's strategic asset allocation globally to a more "liability driven" approach.

This approach integrates well in a surplus optimization framework. In this case, it is clear that the LHP corresponds to the global minimum surplus variance portfolio.

You will notice that, in terms of the mutual funds to be used, the theory only speaks of risk-free asset and zero-beta portfolio, and not of the global minimum variance portfolio. We have found in the literature different approaches to the decomposition of efficient portfolios into LHP and PSP.

We found that the most practical one was proposed by Alex Keel and Heinz H. Müller (1995) from the University of St. Gallen. Let us recall our surplus optimization problem in section 3.3:

$$\min_{x \in \mathbb{R}^n} \left\{ \sum_{i,j} x_i \sigma_{ij} x_j - \frac{1}{f_0} \sum_{i} \sigma_{ij} x_i \right\},$$

with the following constraints:

(A)  \( \sum_{i} x_i = 1, \)

(B)  \( \sum_{i} \mu_i x_i \geq \) some portfolio return \( \mu_p. \)
Let $x^*$ denote the solution of this optimization. Keel and Müller used the Kuhn-Tucker theorem (a generalisation of the Lagrange multipliers, allowing for inequality constraints) to find the optimal conditions:

1. $\sum_y x^* - \frac{1}{f_0} \sum_{ij} - \lambda \mu - \nu e = 0$, 
2. $\mu' x^* = \mu_p$, 
3. $e' x^* = 1$,

where $\lambda \geq 0$ and $\nu \in \mathbb{R}$ are Lagrange multipliers, $\Sigma_y$ is the covariance matrix of the asset returns, $\Sigma_{il}$ is the covariance of the asset returns with the liability return, $\mu$ is the vector of the asset returns, $e$ is a column vector of 1's and 0's is a column vector of 0's.

We obtain the following solution:

$$x^* = \frac{1}{e' \Sigma_{ij}^{-1} e} \Sigma_{ij}^{-1} e + \frac{1}{f_0} \left[ \sum_{ij} - \frac{e' \Sigma_{ij} \sum_{ii}^{il} e}{e' \Sigma_{ij}^{-1} e} \right] + \lambda \left[ \sum_{ij}^{-1} \mu - \frac{e' \Sigma_{ij}^{-1} \mu}{e' \Sigma_{ij}^{-1} e} \right].$$

Computational details are summarized in Appendix C.

Provided that the global minimum surplus variance portfolio $x^{MIN}$ is obtained by omitting constraint (B) from the optimization, this would make $\lambda = 0$, which means that the first part of the equation represents the weights of $x^{MIN}$, as highlighted in the equation below:

$$x^* = \frac{1}{e' \Sigma_{ij}^{-1} e} \Sigma_{ij}^{-1} e + \frac{1}{f_0} \left[ \sum_{ij}^{-1} \sum_{il} - \frac{e' \Sigma_{ij} \sum_{ii}^{il} e}{e' \Sigma_{ij}^{-1} e} \right] + \lambda \left[ \sum_{ij}^{-1} \mu - \frac{e' \Sigma_{ij}^{-1} \mu}{e' \Sigma_{ij}^{-1} e} \right].$$

A further decomposition is even possible, the first term of $x^{MIN}$ does not involve the liabilities at all... it corresponds to the minimum variance portfolio in the absence of liabilities. Keel and Müller propose the following final decomposition:

$$x^* = \frac{1}{e' \Sigma_{ij}^{-1} e} \sum_{ij}^{-1} e + \frac{1}{f_0} \left[ \sum_{ij}^{-1} \sum_{il}^{-1} - \frac{e' \Sigma_{ij} \sum_{ii}^{il} e}{e' \Sigma_{ij}^{-1} e} \right] + \lambda \left[ \sum_{ij}^{-1} \mu - \frac{e' \Sigma_{ij}^{-1} \mu}{e' \Sigma_{ij}^{-1} e} \right].$$
or \( x^* = x^{MIN} + z^{MIN} + \lambda z^* \), where we note that \( z^{MIN} \), the correction stemming from the liabilities is linear in \( \Sigma w \). What does it mean practically in terms of strategic asset allocation? The component \( x^{MIN} \) actually gives the weights of the asset categories in the LHP. We already computed \( x^{MIN} \) in section 3.3 (TAB. 4). They are the weights of the minimum variance portfolios under surplus optimization. Under the no-short sales constraint, let us assume that for the performance seeking, we are comfortable with a maximum risk level of 6%. The corresponding portfolio on the efficient frontier has a return of 5.6%. If we want our overall portfolio to have a return of 5.75%, we will solve the following equation for \( \lambda \):

\[
\mu^t x^* = \mu^t x^{MIN} + \lambda \mu^t z^*.
\]

Since \( \mu_p = 5.75\% \), \( \mu^t x^{MIN} = 4.97\% \) and \( \mu^t z^* = 5.6\% \), we have that \( \lambda = 0.139 \), which means we have the following decomposition (TAB. 5):

<table>
<thead>
<tr>
<th>Weights in LHP ( x^{MIN} )</th>
<th>Weights in PSP ( \lambda z^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>29.95%</td>
<td>0.00%</td>
</tr>
<tr>
<td>10.00%</td>
<td>0.29%</td>
</tr>
<tr>
<td>5.00%</td>
<td>0.57%</td>
</tr>
<tr>
<td>10.00%</td>
<td>2.68%</td>
</tr>
<tr>
<td>10.69%</td>
<td>4.29%</td>
</tr>
<tr>
<td>10.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>12.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>7.13%</td>
<td>4.95%</td>
</tr>
<tr>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>3.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>2.22%</td>
<td>1.12%</td>
</tr>
</tbody>
</table>

TAB. 5: LHP / PSP decomposition of a surplus efficient portfolio with return 5.75%

It turns out that the resulting portfolio is also on the efficient frontier. At this point, you might be asking yourself: if the resulting portfolio is the same that you would have gotten by solving the optimization problem with a return of 5.75%, what is the added value of this decomposition? Its added value lies in its interpretation. First of all, we can think about risk in terms of untradeable risk - the one that is incurred by holding the LHP – and performance
seeking risk, which is the risk that is solely dedicated to enhancing the performance. The idea is that the LHP is the portfolio that we have to always hold on to and that the PSP is what we change. If our return objective is 5.75%, and after investing in the LHP, we have little wealth left, we will choose to invest in a high return and high risk PSP ($\lambda$ is small) as in our example above. If we have a lot of wealth left and do not want to take on too high of a risk, we will invest in a low return and low risk PSP ($\lambda$ is large): hence the designation of dynamic or modern ALM, which hints at the variability of the PSP. We'll throw in another three-letter word: LDI -Liability Driven Investment, which in essence means that liabilities are taken into account in the choice of investment instruments\(^8\), but whose meaning, in general, has been brought down to the practice of holding an LHP and a PSP.

This method being essentially the surplus optimization method, the difficulties encountered in its implementation are the same. Furthermore because this is a decomposition that was derived using basic optimization constraints (under which, negative asset weights are actually possible), it is probable that the decomposition will not always work depending on the choice of $\gamma^\prime$. The stricter the constraints are, the lesser combinations are possible.

Carrying out this research, we found different ways of obtaining the LHP and the PSP. The most common has been the "manual" construction of the liability hedging portfolio, which means matching the bonds to the liability payments. The resulting portfolio of bonds is considered to be the LHP. Often swaps are then used to synthesize the bonds. The problem with this is that the correlations with other assets are not being considered. The surplus optimization finds the optimal asset mix that manages to keep the surplus risk at zero, whereas the manual construction method just finds the liability mimicking portfolio. When the LHP is manually constructed, introducing it back into the framework of efficient portfolios becomes tedious: the basic assumptions are that only risky assets are considered for the PSP and that the LHP is not correlated with those risky assets. In other words, we end up being overly conservative with the hedging of the liabilities and overly reckless with the performance seeking risk (see reference [8] for more details on this methodology). The same authors suggest another simplification of this methodology, which consists of considering the pension fund is closed, because "the sponsor may arbitrarily close or modify the pension plan for newcomers". Some other LDI practitioners have echoed this suggestion, hence giving LDI another 3-letter synonym, TDI (Termination Driven Investment). TDI is again extremely

\(^8\) This would also qualify surplus optimization as LDI.
conservative in the sense that new contributions are not accounted for, hence the investment decisions may not make much of a difference in the short-term, but often turn out not to be very optimal in the long-run (see [13] for a detailed example of the effect of TDI on the efficient frontier).

Lastly, we also have to underline the fact that the LHP/PSP method as well as all the other methods mentioned above has a one-period framework. We are doing an optimization from time 0 to time 1, which actually is 10 years in our model. We chose this approach because of the fact that it does not diverge from the reasoning pension funds currently use in their investments, which made the underlying concepts easier to grasp. However, continuous-time models of asset–liability management are out there. In particular that of Martellini (2006) comes to mind, where stochastic processes model the prices of the assets and the present value of the liabilities. Martellini assumes that a risk-free asset with return \( r \) in the form a default-free bond is traded in the economy. We end up with an optimal portfolio solution, which is a function of time and of the generator of the stochastic process followed by the funding ratio. This leads to an entirely different decomposition, described as the three-fund separation theorem:

\[
- w_M = \frac{\left(\Sigma_y \Sigma_y'\right)^{-1} \cdot (\mu - r \cdot e)}{e' \cdot \left(\Sigma_y \Sigma_y'\right)^{-1} \cdot (\mu - r \cdot e)},
\]

which are the asset weights for an asset-only mean-variance portfolio,

\[
- w_L = \frac{\left(\Sigma_y'\right)^{-1} \cdot \Sigma_{yl}}{e' \cdot \left(\Sigma_y'\right)^{-1} \cdot \Sigma_{yl}},
\]

which are the assets weights for the portfolio with the highest correlation with the liabilities,

- the rest being invested in the risk free asset.

The obvious challenge in this three-separation theorem is the availability of a risk free asset. However, this is an interesting lead in terms of dynamic optimization: the weights in each of those portfolios depend on the generator of the stochastic process followed by the funding ratios.

The purpose of section 3 was to identify a few portfolios of interest, which match the risk tolerance of the pension funds and their desired level of performance. We have seen two main methods: asset-only and surplus optimizations, although variations of those methods abound. The value of our assets for each of the next 10 years depends on the performance of the
chosen portfolios. In section 2, we also set up a model to value the liabilities for each of the next 10 years. These models were built on our convictions of what will happen in the future: in particular they rely heavily on economic assumptions made over a period of 10 years. While we hold on to these long-term projections as valid, our models and entire analysis depending on those projections, we must also bear in mind the "unpredictable" behaviour of economic parameters on a year to year basis. In other words the mere knowledge of the average return of an asset over the past 10 years does not necessarily give me a clear idea of what its return was three years from now. To account for the variability of the parameters on a year to year basis, we will simulate various economic scenarios, under which we will compute on one side the value of the liabilities and on the other side the value of the assets, depending on whether we are holding portfolio 1, portfolio 2 and so on ... For each of these scenarios and each of these portfolios, we will compute the funding ratios resulting from the evaluation of the assets and the liabilities. The funding ratio results are collected for each of these portfolios; the one deemed as having the best combination of performance, risk level and low value at risk for the portfolio is chosen as the strategic allocation of the pension fund. Section 4 will focus on the simulation of the economic scenarios while section 5 will focus on the final analysis and the choice of the strategic asset allocation.

4 Simulation of the financial parameters

The purpose of using simulation to generate economic scenarios is to analyze the position of the pension fund under all possible realistic economic situations, from the very best to the very worst ones on a year to year basis. This is a crucial step in that we get to see the possible short term effect of our long term planning.

In practice, this means that we start out with a vector of the values of the financial parameters today. This vector is represented by the first node in FIG. 2 below. Let us say that our ALM study will be for the next 10 years and that we have chosen to simulate 200'000 scenarios. We find the second node by modelling the parameters for the following year, then the third node and so on. Once we find the 10th node, we have completed one economic scenario (highlighted in green), and we can move on to simulate the 199'999 remaining ones.
A possible state of the economy over the next 10 years

FIG. 7: Linear scenario generation

The question has now become: how do we get from one node to another? Again, as with many other aspects of ALM, there is not one single "right" way to achieve this. The field of econometrics offers many models to choose from, each with its advantages and drawbacks. The choice of model depends on whether we use a same-level or top down approach to the interpretation of the relationship between the parameters and on making sure that the results do pass the plausibility test of the pension fund.

4.1 Interpretation of the relationship between the parameters

In section 1.2, we listed the financial parameters often taken into consideration in the context of an ALM and pointed briefly at the relationship between them, which we illustrate in FIG. 8.

First let us note the addition of the yield curve, which is basically the plot at a set point in time of the interest rates of zero coupon bonds with varying maturity dates. The 10-year bond interest rate at each point in time can be used as the technical interest rate and the varying maturity bond interest rates can be used to compute the predicted returns of the bonds in our portfolio.

Furthermore, the salary growth rate is included as being correlated with inflation, but of course other factors such as seniority will have to be taken into account. The contribution rate in Switzerland usually depends on the age of the insured. However many Swiss pension funds specify in their rulebook the right to increase or decrease the contribution rate of their insured depending on the level of their funding ratio. We included the contribution rate in the figure because it can be affected by the other variables if they globally lead to especially high or low
funding ratios. To best capture the reality of the pension fund, these two parameters are often entered as step functions depending on the age of the insured, inflation, funding ratio level of the previous year, etc... The models described in this section focus on the simulation of the remaining parameters.

![Diagram showing the relationship between financial parameters in the context of an ALM](image)

**FIG.8: Relationship between financial parameters in the context of an ALM**

### 4.2 The plausibility test

When we choose a model, we already have in mind a general idea of the possible behaviour of each economic variable. For example some variables always revert to the mean, so a model that will have those variables jump from really low to really high values in consecutive periods of time will not be suitable. Additionally, the purpose of simulation is not to consider every single scenario under the sun. Doing so would render unusable results: they would be all over the place. We know that when the economy is looking up, we benefit from it and when the economy is looking down, we suffer because of it. The idea is thus not to consider extreme cases, if those would annihilate the 10-year asset return assumptions that we used as inputs to the optimization problem. If we were to fall into such extreme cases, any of the chosen portfolios would no longer be a suitable strategic allocation. Let us use a simple weather analogy: think of it as making plans for sunny weather. You might still enjoy your day despite the occasional light rain, but it takes one thunderstorm to ruin your plans. We are here to find out how well a portfolio can withstand "likely" unfavourable conditions. Again
with the weather analogy: if a thunderstorm is considered "likely", why prepare for sunny weather? This is what we mean by the plausibility test: the behaviour of the economic variable from one period to another should be realistic and there should be a certain continuity between the assumptions used in the construction of the portfolio and the simulation of the economic scenarios. If the process or model we choose fail our plausibility test, we need to think about whether it needs to be recalibrated, whether our assumptions need to be reevaluated or whether we need to use a different model all together.

4.3 Simulation methodology

Monte-Carlo simulation consists of randomly assigning values to each of the variables. The value set for those variables can be its historical records (bootstrapping), an assumed distribution (e.g. independent drawings from a multivariate normal distribution) or results of processes generating those variables (e.g. AR, VAR, ARMA, GARCH). Our approach to modelling those variables also depends on whether or not we assume a causality relationship between the parameters. If we do not, we can just use one model to generate all variables – let the correlation between the variables and the characteristics of the chosen model define the relationship between the parameters. Otherwise, to reflect the assumed causality, we use a top-down approach, which consists of first modelling the independent variables such as inflation, then using them as input in the model of the remaining dependent variables. Wilkie's model and many consultants' private models follow a top-down approach. We will summarize a few of those models and processes just to get a general idea of the procedure behind the simulation of economic scenarios. This is important as we become aware of the inherent assumptions that come with our choice of modelling, which adds an extra dimension to the decision making process. However, we refer the reader to the literature (see [7], [14], [18] and [26]) for a more scientific development on the topic.

4.3.1 Bootstrapping

Bootstrapping consists of drawing random samples directly from the historical records. Let us say that we have past monthly records for each of the variables. Since we want to generate scenarios for a long horizon, we would need at least 120 monthly returns for different points in time. Every time we pick a random date, the entire sample of returns for the previous month is used. This way, we do preserve the correlations between the variables. After picking
12 random dates, we compound the 12 monthly returns: this is one scenario of the 10-year return (i.e. one node of FIG. 7). We use the same process to obtain the other nodes. With this method, variables do not have to be assigned a given distribution. However there is no mean reversion, i.e. parameters can jump from having high values in one year to low values the following year, or remain low or high for a long time, which is not realistic for some parameters.

4.3.2 Independent drawings from a multivariate normal distribution

In this method, we assume that the variables follow a multivariate normal distribution and we

In order to use this model, you will need to factor the covariance matrix $\Sigma$ using the Cholesky decomposition such that $\Sigma = CC^T$.

The future returns for each asset are then obtained with the following equation:

$$Y_i(t+1) = Y_i(t)e^{\mu_i \sum_{j \neq i} c_{ij} \epsilon_{j,t+1}},$$

where $Y_i(t)$ and $Y_i(t+1)$ are respectively the yield rates of asset $i$ at time $t$ and $t+1$, $\mu_i$ is the expected yield rate of asset $i$, $c_{ij}$ is the element $i, j$ of the matrix $C$ found above and $\epsilon_{j,t+1} \sim N(0,1)$.

This model is very simple to implement and is also stationary provided that the parameters are chosen in the right range. Its major drawback lies in its constraint that all variables follow a normal distribution, which is not necessarily plausible for variables such as inflation.

4.3.3 AR and VAR processes

An Auto-Regressive process (AR) is a regression over the historical values of a variable. The future returns are obtained with the following equation:

$$Y_{t+1} = c + \varphi_1 Y_t + \ldots + \varphi_n Y_{t-n} + \epsilon_{t+1},$$

where $c = \mu \cdot (1-\varphi_1-\ldots-\varphi_n)$, and where $Y_t$ and $Y_{t+1}$ are respectively the yield rates of the variable at time $t$ and $t+1$, the $\varphi$’s are regression parameters, $n$ is the order of Auto-Regression and $\epsilon_{j,t+1} \sim N(0,1)$. 


A Vector Auto-Regressive process (VAR) is the generalization of AR to the multivariate case. It is a regression over not only the historical values of variable $i$, but also over all the historical values of the other parameters.

Vector autoregressive processes are often used in the ALM context. First of all, they are stationary, which means that the variance and expected values remain constant over time. The process also satisfies the mean reversion property. The fact that it is not memoryless, in that it takes into account the full evolution of the variable before a given time $t$ makes it a suitable choice for inflation and interest rates. More importantly it integrates well with models such as the Nielsen Siegel model, used to build yield curves. Historical data of the inputs of the Nielsen Siegel model – long interest rate, short interest rate, curvature and scaling parameters – can be fed into the VAR model along with the other economic quantities. A yield curve is then built for each set of outputs from the process and desired interest rates for this scenario will be drawn from this modelled yield curve.

We can go on with this model does this, this model does that, but there are simply too many choices out there. Those proposed by consultant companies such as Tower Watson are difficult to analyze because of their proprietary nature. The examples above wished to provide some sort of direction in terms of model characteristics to examine. At the end of the day, all involved in the ALM process need to feel confident in the chosen model's predictive capacity. These scenarios serve the important purpose of forecasting how our eventual strategic asset allocation will hold up on a year to year basis. This forecast is only as meaningful as the credibility of the basis on which we are making it.

5 Results

For each of the scenarios found in section 4, we can compute the possible values of the assets (based on portfolios from section 3) and the value of the liabilities (based on the liability model from section 2). Often benchmarks such as LPP-40 are also included in the scenario tests to compare their performance with that of our chosen portfolios. Therefore, if we simulated 200'000 economic scenarios, we would end up with 200'000 resulting funding ratios for each of those portfolios per year (see FIG. 9). Let us note that all the examples presented in this section are purely theoretical.
5.1 The funding ratio analysis

Viewing the results in terms of percentiles facilitates their interpretation and gives a real factor of comparison between the portfolios. Often the board of trustees is interested in the following aspects:

1. What is the spread of the funding ratio over the years for each of the portfolio?

We take the results obtained by a given portfolio for each year and calculate the same percentiles for each set of data (see FIG. 10).
2. What is the probability of shortfall for each portfolio over the years?

We compute the ratio of the number of scenarios leading to funding ratios less than 100% over the total number of scenarios, in this case 200'000 scenarios, for each set of data (see FIG. 11)

![FIG.11: Probability of shortfall for our choices of portfolio](image)

3. In the event of underfunding, what is the average funding ratio under each portfolio?

We narrow down our funding ratios to only those less than 100%, out of which we calculate the median (see FIG. 12).

![FIG.12: Median funding ratio under the condition of shortfall](image)

Other comparative figures can be generated depending on the type of information the board of trustees needs to make its final decision. In the end, the portfolio that best satisfies the risk tolerance and return expectations of the pension fund, while exhibiting acceptable funding
ratio behaviour will be adopted as its strategic asset allocation. How does this translate into investment decisions?

### 5.2 The tactical asset allocation assignment

Each pension fund presents its asset managers with the strategic asset allocation as the ideal investment portfolio. However, when the situation calls for it, asset managers have the "flexibility" of moving away from this allocation at their discretion, as long as they remain within certain upper and lower bounds for each asset category. Those so-called leeways are generally very narrow, so as to avoid for the pension fund to get out of course. We did not find much in the literature as to how those leeways are determined, but it would only make sense that they should reflect the constraints put on the asset categories during the ALM analysis. Looking at the behaviour of the benchmarks and other asset allocations candidates can be a good starting point in determining those leeways. Nonetheless, portfolios at both ends of the leeways will have to be tested under the scenario analysis to verify their potential impact on the pension fund's funding ratio.

disponibilité
PART II: Matching expectations to the ALM model characteristics

Pension funds face a double challenge: they must ensure that their liabilities will be covered and that performance will be generated. These are two sides of the same coin: in order for pension funds to honour their liabilities, they need to generate performance, which comes at a risk - the type of risk that can jeopardize a pension fund's ability to cover its liabilities. The solution: Strike the "right balance", right?

It may seem that pension funds are looking for the "right balance" in implementing their ALM studies. Intuitively, this would mean that they would calculate the needed portfolio return to ensure a funding ratio of 100% - or some realistically achievable funding ratio level, if the pension fund is severely underfunded. We will call this portfolio the survival return portfolio. This return is often calculated by pension funds, but how many actually hold the survival return portfolio as their strategic asset allocation? The answer is probably none. Pension funds often end up choosing the portfolio with the highest return that matches some risk tolerance arguably reflecting their wealth, current funding situation and structure of liabilities.

In this sense, we are very far from the balance paradigm: ALM studies, as we know them, are very much performance driven, matching the maximum return possible to the maximum risk tolerance of the pension fund. This preference is reflected not only in the portfolio choice but also in the assumptions and the ALM methodology itself. Certain long-term return assumptions simply imply that certain shocks are not accounted for, which we reinforce through our scenario analysis. Such ALM practice is not wrong at all; under "normal" economic conditions, the ALM study provides the pension fund with much needed direction. Under crisis situations, the whole structure crumbles, as the founding assumptions are no longer valid.

When the economic crisis hit in 2008, some pension funds held on to a false sense of security by keeping their strategic asset allocation, perhaps reassured by the memory of descending graphs of funding ratio probabilities. Others clearly understood that their ALM was no longer valid, but kept their allocation just the same for lack of an alternative. This should not have been the case since most pension funds, in their quest for performance superior to the survival return had taken on unnecessary risk exposure, which at the first sign of troubled times, they should have gotten rid of. This does not mean that they would have escaped the consequences of the crisis, but they would have been less affected and their risk exposure would conform their profiles.
Looking at the situation this way brings to mind the LDI framework: the breakdown of the strategic asset allocation into a Liability Hedging Component and a Performance Seeking component. While this is by no-means a recognition of LDI as a new, state-of-the-art ALM replacement, it is certainly an acknowledgment that we need to move away from a performance-centric approach. With the long time horizon of their liabilities, it is understandable that pension funds are diligent about delivering the performance, but exactly because of this long time horizon, they are particularly sensitive to economic crises. Where one mismanaged crisis in its lifetime can have dire consequences for its members, it is not hard to imagine what two or more crises can do. Hence, the need for a multi-level approach to investment strategy. LDI achieved this layering quite elegantly, by placing it in the optimization framework, which ensured the efficiency of our investment strategy. However, for lack of available instruments and of cash, it has become the poster agent for little understood derivatives such as interest rate swaps. The bottom line idea of investment building blocks however remains relevant, and while holding a Liability Hedging Portfolio is simply not conceivable by many pension funds, and swaps seem by far to not be a viable solution to the problem, we can still build upon what we know from LDI to come up with our own building blocks. This approach changes the role of our ALM. In this perspective, there would be some changes to the methodology as well. In the following table (TAB. 6), we highlight the points we need to pay attention to at each stage of the ALM process and the challenges that we might face.

1. Assumptions

Our financial assumptions regarding the returns should neither pessimistic nor optimistic. They should simply be in agreement with the convictions of the board of trustees and of the investment committee.

The reasons behind those convictions should be carefully documented (historical, views on the future behaviours, etc...).

If historical, we should also document which indices have been used.

2. Modelling of the assets

A surplus framework should be considered, even if that means that constraints on certain asset categories have to be made to reflect the pension fund's investment limitations. Since
the biggest drawback of this method is the availability of certain investment instruments, the constraints will provide for the best solution under realistic investment conditions, even if this means that the asset liability mismatch risk due to interest rate changes is not hedged. This method still provides the advantage of including this risk factor in the decision making process and of being able to monitor this risk.

3. Scenario simulation

Crises observations will also be included in the simulation of future asset returns. Different investment strategies can be tested to compare which serves best the interests of the pension fund. In his master thesis “Pension funds – Prepare for crisis!”, Marco Van der Lans (2011) proposes a model that introduces crises observations into the simulation with the use of copulas. Bram Masselink (2009) also proposes in his master thesis “Pension funds and economic crises” a methodology that incorporates a possible economic crisis in the VAR model using spectral analysis. We see that most efforts for pension funds to incorporate economic crises in their analysis have concentrated in this area because of the anchored view that the strategic asset allocation must be static. While including economic crises might bias the choice of strategic asset allocation, in the end we have a portfolio that is supposed to work under both normal and extreme conditions. Investment portfolios do not make miracles and there will be a trade-off between so-called damage control during economic crisis and performance during normal conditions. The multi – level investment solution proposes better damage control and better performance generation with the dynamic character of the strategic asset allocation.

TAB. 6: Description of the impact of the multi-level investment approach on ALM steps

So far, pension funds saw in ALM studies a way to keep an eye on the future and were comforted in having this one direction to follow. If the ALM turns so dynamic, then what is the point of doing it at all? Why not just invest as we see fit based on our convictions? This seems to bring the whole existence of ALMs into question, but when you embark on a long voyage like the ones that pension funds take on, you are always better off having an itinerary and you will be even better off if you have action plans ready for inevitable disturbances. In that regard, ALM is every bit the risk management process described in the definition provided by the Society of Actuaries (see introduction).
CONCLUSION

This research paper came to be due to the disillusionment of Swiss pension funds with their ALM after the economic crisis hit in 2008. We broke down the ALM process to bridge the clear gap between the mechanisms of ALM and the interpretation of its results by pension funds. In the process, we were able to point out certain inconsistencies with the way the process is carried out, in particular with the use of the asset-only model during the mean-variance optimization. By going over the various methodologies found in the literature and in practice, we hope to have provided institutional key players with a lens through which to evaluate the robustness of their ALMs and to make sound interpretations of the obtained results. This research would have been incomplete without the analysis of a so-called new, state-of-the-art methodology to “replace” ALMs, Liability Driven Investment, which at best is an extension of ALM, as we know it. However, even with questionable implementation through the use of swaps, LDI was a springboard in terms of getting us to think of the strategic asset allocation differently, i.e. as a multi-level investment strategy. Admittedly, the construction of such a portfolio was not attempted, but one would imagine it to be specific to the pension fund’s interest in hedging the asset liability mismatch risk, comfort level when it comes to holding cash, and risk tolerance on the performance seeking components of the portfolio. The challenges that come with such a portfolio construction is the question of timing in terms of when to trim/add the performance seeking component(s) of the portfolio, and the substantial freedom left to investors. To put those challenges into perspective, let us make an analogy between pension funds and planes. No matter how reliable the autopilot is, you are glad to have quick thinking pilots at the commands, and when the turbulence cannot be avoided, the pilots need the necessary room for manoeuvre to put that quick thinking to good use.

We have seen that the ALM process calls on areas such as actuarial science, financial theory and econometrics, which are constantly evolving. Certain of these developments have made it into the ALM configuration, at least theoretically, with the use for example of the Jorion-Stein optimization method to involve higher moments in the portfolio decision making process. This leaves a lot of room for practical improvements and as such, ALM practitioners need to keep abreast those developments in the best interest of their clients, the pension funds and intrinsically that of the pension plan members.
APPENDIX A

Assumptions asset categories for the asset–only model

<table>
<thead>
<tr>
<th>Asset Categories</th>
<th>Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>UBS Money Market Fund</td>
</tr>
<tr>
<td>Bonds CHF</td>
<td>SBI Domestic, total return</td>
</tr>
<tr>
<td>Foreign Bonds CHF</td>
<td>SBI Foreign, total return</td>
</tr>
<tr>
<td>Mortgages</td>
<td>SBI Domestic, total return</td>
</tr>
<tr>
<td>Bonds FOREX</td>
<td>Citigroup World Govt Bond Index, total return</td>
</tr>
<tr>
<td>Equity CHF</td>
<td>Swiss Performance Index</td>
</tr>
<tr>
<td>Equity World</td>
<td>MSCI World Equities Index, total return</td>
</tr>
<tr>
<td>Real estate CHF</td>
<td>SWX Real total return Index</td>
</tr>
<tr>
<td>Swiss real estate fdp</td>
<td>SWX Real total return Index</td>
</tr>
<tr>
<td>Foreign real estate fdp</td>
<td>GPR 250 World USD</td>
</tr>
<tr>
<td>Hedge funds</td>
<td>HFR Global Index</td>
</tr>
<tr>
<td>Commodities</td>
<td>Rogers Int. Comm. Index, total return</td>
</tr>
</tbody>
</table>

Table A. 1: Reference indices for asset categories

<table>
<thead>
<tr>
<th>Asset Categories</th>
<th>Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>1.50%</td>
<td>0.20%</td>
</tr>
<tr>
<td>Bonds CHF</td>
<td>2.80%</td>
<td>2.90%</td>
</tr>
<tr>
<td>Foreign Bonds CHF</td>
<td>3.30%</td>
<td>3.30%</td>
</tr>
<tr>
<td>Mortgages</td>
<td>3.80%</td>
<td>2.90%</td>
</tr>
<tr>
<td>Bonds FOREX</td>
<td>4.40%</td>
<td>10.20%</td>
</tr>
<tr>
<td>Equity CHF</td>
<td>7.30%</td>
<td>16.90%</td>
</tr>
<tr>
<td>Equity World</td>
<td>7.30%</td>
<td>20.70%</td>
</tr>
<tr>
<td>Real estate CHF</td>
<td>4.50%</td>
<td>4.00%</td>
</tr>
<tr>
<td>Swiss real estate fdp</td>
<td>5.30%</td>
<td>7.10%</td>
</tr>
<tr>
<td>Foreign real estate fdp</td>
<td>5.50%</td>
<td>18.80%</td>
</tr>
<tr>
<td>Hedge funds</td>
<td>4.80%</td>
<td>13.00%</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.60%</td>
<td>20%</td>
</tr>
<tr>
<td>Other</td>
<td>4.80%</td>
<td>13.00%</td>
</tr>
</tbody>
</table>

Table A. 2: Return and risk statistics for asset categories
<table>
<thead>
<tr>
<th>CORRELATIONS</th>
<th>Cash</th>
<th>Bonds CHF</th>
<th>Foreign Bonds CHF</th>
<th>Mortgages</th>
<th>Bonds FOREX</th>
<th>Equity CHF</th>
<th>Equity World</th>
<th>Real estate CHF</th>
<th>Swiss real estate fdp</th>
<th>Foreign real estate fdp</th>
<th>Hedge funds</th>
<th>Commodities</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>1</td>
<td>0.017</td>
<td>0.003</td>
<td>0.017</td>
<td>0</td>
<td>0.003</td>
<td>-0.032</td>
<td>-0.02</td>
<td>0.018</td>
<td>-0.013</td>
<td>-0.012</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bonds CHF</td>
<td>0.017</td>
<td>1</td>
<td>0.767</td>
<td>1</td>
<td>0.394</td>
<td>-0.361</td>
<td>-0.31</td>
<td>-0.161</td>
<td>-0.017</td>
<td>-0.196</td>
<td>-0.207</td>
<td>0</td>
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<td>1</td>
<td>0.767</td>
<td>0.303</td>
<td>-0.127</td>
<td>-0.096</td>
<td>-0.025</td>
<td>0.054</td>
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<td>0.394</td>
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<td>-0.31</td>
<td>-0.161</td>
<td>-0.017</td>
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<td>-0.211</td>
<td>-0.039</td>
<td>0.009</td>
<td>0.03</td>
<td>-0.016</td>
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<td>-0.361</td>
<td>-0.262</td>
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<td>0.719</td>
<td>0.373</td>
<td>0.103</td>
<td>0.468</td>
<td>0.418</td>
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<td>-0.096</td>
<td>-0.31</td>
<td>-0.211</td>
<td>0.719</td>
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<td>0.123</td>
<td>0.682</td>
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<td>0.339</td>
<td>0</td>
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<td>-0.025</td>
<td>-0.161</td>
<td>-0.039</td>
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<td>0.352</td>
<td>0.298</td>
<td>0.189</td>
<td>0</td>
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<tr>
<td>Swiss real estate fdp</td>
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<td>-0.017</td>
<td>0.054</td>
<td>-0.017</td>
<td>0.009</td>
<td>0.103</td>
<td>0.123</td>
<td>0.162</td>
<td>1</td>
<td>0.131</td>
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<td>0.03</td>
<td>0.468</td>
<td>0.682</td>
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<td>0.298</td>
<td>0.08</td>
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<td>-0.002</td>
<td>-0.147</td>
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<td>0.339</td>
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<td>0.33</td>
<td>0.379</td>
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<td>0</td>
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Table A. 3: Correlations for asset categories
APPENDIX B

Assumptions asset and liabilities categories for the surplus optimization model

<table>
<thead>
<tr>
<th></th>
<th>Risk</th>
<th>Liability-accrued benefit</th>
<th>Liability-wage inflation</th>
<th>Liability-wage growth</th>
<th>Equity</th>
<th>Nominal bonds</th>
<th>Real bonds</th>
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<tbody>
<tr>
<td>Liability-accrued benefit</td>
<td>0.08</td>
<td>1</td>
<td>0.93</td>
<td>0.48</td>
<td>0.31</td>
<td>0.99</td>
<td>0.83</td>
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<td>Liability-wage inflation</td>
<td>0.099</td>
<td>0.93</td>
<td>1</td>
<td>0.42</td>
<td>0.24</td>
<td>0.87</td>
<td>0.97</td>
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<tr>
<td>Liability-wage growth</td>
<td>0.198</td>
<td>0.48</td>
<td>0.42</td>
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<td>0.98</td>
<td>0.50</td>
<td>0.37</td>
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<tr>
<td>Equity</td>
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<td>0.24</td>
<td>0.98</td>
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<td>0.33</td>
<td>0.22</td>
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<td>Nominal bonds</td>
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<td>0.33</td>
<td>1</td>
<td>0.76</td>
</tr>
<tr>
<td>Real bonds</td>
<td>0.026</td>
<td>0.83</td>
<td>0.97</td>
<td>0.37</td>
<td>0.22</td>
<td>0.76</td>
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TAB. B.1: Risk and correlation of liability components and asset categories

<table>
<thead>
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<th></th>
<th>Weights</th>
<th>Risk</th>
<th>Liability-accrued benefit</th>
<th>Liability-wage inflation</th>
<th>Liability-wage growth</th>
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</thead>
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<td>Liability-accrued benefit</td>
<td>0.8</td>
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<td>1</td>
<td>0.93</td>
<td>0.48</td>
</tr>
<tr>
<td>Liability-wage inflation</td>
<td>0.1</td>
<td>0.099</td>
<td>0.93</td>
<td>1</td>
<td>0.42</td>
</tr>
<tr>
<td>Liability-wage growth</td>
<td>0.1</td>
<td>0.198</td>
<td>0.48</td>
<td>0.42</td>
<td>1</td>
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</tbody>
</table>

\[ \sigma_{\bar{\kappa}} = 8.45\% \]

TAB. B.2: Recombining the decomposed liability
<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Correlations with $\tilde{r}_i$</th>
<th>Covariances with $\tilde{r}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
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<tr>
<td>Bonds CHF</td>
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<td>0.00228</td>
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<tr>
<td>Foreign Bonds CHF</td>
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<td>0.00223</td>
</tr>
<tr>
<td>Mortgages</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bonds FOREX</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Equity CHF</td>
<td>0.37</td>
<td>0.00529</td>
</tr>
<tr>
<td>Equity World</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Real estate CHF</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Swiss real estate fdp</td>
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<td>0</td>
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<tr>
<td>Foreign real estate fdp</td>
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<td>0</td>
</tr>
<tr>
<td>Hedge funds</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Commodities</td>
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</tr>
<tr>
<td>Other</td>
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<td>0</td>
</tr>
</tbody>
</table>

TAB. B.3: Correlations and covariances of each asset class with $\tilde{r}_i$
APPENDIX C

Deriving the optimal portfolio in the surplus optimization model algebraically

We start out with the surplus optimization problem, which is expressed as:

$$\min_{x \in \mathbb{R}^n} \left\{ \sum_{i,j} x_i \sigma_{ij} x_j - \frac{1}{f_0} \sum_{i \in \mathbb{C}} \sigma_{ii} x_i \right\},$$

with the following constraints:

(C) \quad \sum_{i \in \mathbb{C}} x_i = 1,

(D) \quad \sum_{i \in \mathbb{C}} \mu_i x_i \geq \text{some portfolio return } \mu_p.

Let \( x^* \) denote the solution of this optimization. With the Kuhn /Tucker theorem (a generalisation of the Lagrange multipliers, allowing for inequality constraints), we know that we have to satisfy the following optimal conditions:

1. \( \sum_{ij} x^* - \frac{1}{f_0} \sum_{il} -\lambda \mu - ve = 0 \),
2. \( \mu' x^* = \mu_p \),
3. \( e' x^* = 1 \),

where \( \lambda \geq 0 \) and \( v \in \mathbb{R} \) are Lagrange multipliers.

First, we rearrange condition (1):

\( \sum_{ij} x^* - \frac{1}{f_0} \sum_{il} -\lambda \mu - ve = 0 \).

Then we multiply both sides by \( \sum_{ij}^{-1} \):

\( x^* = \frac{1}{f_0} \sum_{ij}^{-1} \sum_{il} -\lambda \sum_{ij}^{-1} \mu - v \sum_{ij}^{-1} e \).

Again, let us multiply both sides by \( e' \):

\( e' x^* = \frac{1}{f_0} e' \sum_{ij}^{-1} \sum_{il} -\lambda e' \sum_{ij}^{-1} \mu - ve' \sum_{ij}^{-1} e \).

Now substituting in condition (3), we obtain:

\[ 1 = \frac{1}{f_0} e' \sum_{ij}^{-1} \sum_{il} -\lambda e' \sum_{ij}^{-1} \mu - ve' \sum_{ij}^{-1} e. \]

We have to isolate \( v \):

\[ v = \frac{1}{e' \sum_{ij}^{-1} e} \left[ 1 - \frac{1}{f_0} e' \sum_{ij}^{-1} \sum_{il} -\lambda e' \sum_{ij}^{-1} \mu \right]. \]

Then replace \( v \) back in condition (1):
\[ x^* = \frac{1}{f_0} \sum_{ij}^{-1} \sum_{iL} - \lambda \sum_{ij}^{-1} \mu - \frac{1}{e'} \sum_{ij}^{-1} e \left[ 1 - \frac{1}{f_0} e' \sum_{ij}^{-1} \sum_{iL} - \lambda e' \sum_{ij}^{-1} \mu \right] \sum_{ij}^{-1} e. \]

After rearranging the terms, we finally obtain:

\[ x^* = \frac{1}{e' \sum_{ij}^{-1} e} \sum_{ij}^{-1} e + \frac{1}{f_0} \left[ \sum_{ij}^{-1} \sum_{iL} - \frac{e' \sum_{ij} \sum_{iL}}{e' \sum_{ij}^{-1} e} \sum_{ij}^{-1} e \right] + \lambda \left[ \sum_{ij}^{-1} \mu - \frac{e' \sum_{ij}^{-1} \mu \sum_{ij}^{-1} e}{e' \sum_{ij}^{-1} e} \right]. \]
REFERENCES


